7-5 Exponential and Logarithmic Equations

Objective To solve exponential and logarithmic equations

Lesson Vocabulary
- exponential equation
- logarithmic equation

Any equation that contains the form b^{cx}, such as $a = b^{cx}$ where the exponent includes a variable, is an **exponential equation**.

Essential Understanding You can use logarithms to solve exponential equations. You can use exponents to solve logarithmic equations.

Problem 1 Solving an Exponential Equation—Common Base

Multiple Choice What is the solution of $16^{3x} = 8$?

(A) $x = \frac{1}{4}$ (B) $x = \frac{3}{7}$ (C) $x = 1$ (D) $x = 4$

Plan

What common base is appropriate?
2 because 16 and 8 are both powers of 2.

$16^{3x} = 8$

$(2^4)^{3x} = 2^3$ Rewrite the terms with a common base.

$2^{12x} = 2^3$ Power Property of Exponents

$12x = 3$ If two numbers with the same base are equal, their exponents are equal.

$x = \frac{1}{4}$ Solve and simplify.

The correct answer is A.

Got It? 1. What is the solution of $27^{3x} = 81$?

When bases are not the same, you can solve an exponential equation by taking the logarithm of each side of the equation. If m and n are positive and $m = n$, then $\log m = \log n$.

Problem 2 Solving an Exponential Equation—Different Bases

What is the solution of $15^{3x} = 285$?

Think

Which property of logarithms will help isolate x?
The rule $\log a^x = x \log a$ moves x out of the exponent position.

$$15^{3x} = 285$$

$$\log 15^{3x} = \log 285 \quad \text{Take the logarithm of each side.}$$

$$3x \log 15 = \log 285 \quad \text{Power Property of Logarithms.}$$

$$x = \frac{\log 285}{3 \log 15} \quad \text{Divide each side by 3 log 15 to isolate } x.$$

$$x \approx 0.6958 \quad \text{Use a calculator.}$$

Check $15^{3x} = 285$

$$15^{3(0.6958)} \approx 285.0840331 \approx 285 \checkmark$$

Got It? **2. a.** What is the solution of $5^{2x} = 130$?

b. Reasoning Why can't you use the same method you used in Problem 1 to solve Problem 2?

Problem 3 Solving an Exponential Equation With a Graph or Table

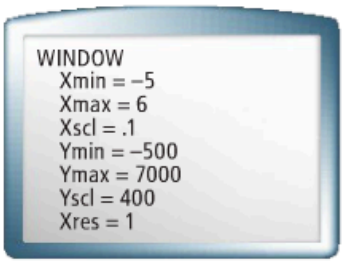
What is the solution of $4^{3x} = 6000$?

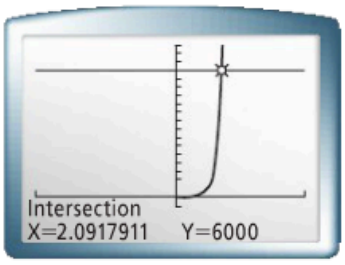
Method 1 Solve using a graph.

Use a graphing calculator. Graph the equations.

$$Y_1 = 4^{3x}$$
$$Y_2 = 6000$$

Adjust the window to find the point of intersection. The solution is $x \approx 2.09$.





Method 2 Solve using a table.

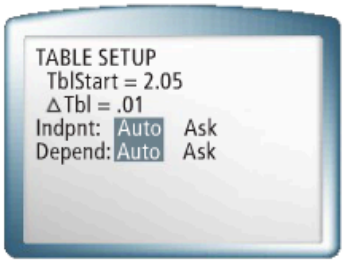
Think

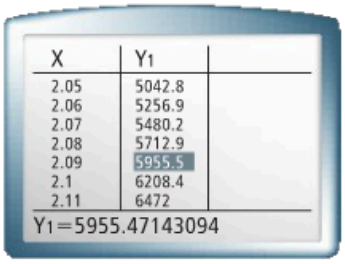
How do you choose TblStart and ΔTbl values?
Start with 0 and 1, respectively. Adjust both values as you close in on the solution.

Use the table feature of a graphing calculator. Enter $Y_1 = 4^{3x}$.

Use the **TABLE SETUP** and **ΔTbl** features to locate the x-value that gives the y-value closest to 6000.

The solution is $x \approx 2.09$.





Got It? **3.** What is the solution of each exponential equation? Check your answer.

a. $7^{4x} = 800$ **b.** $5.2^{3x} = 400$

Problem 4 Modeling With an Exponential Equation

Resource Management **Wood is a sustainable, renewable, natural resource when you manage forests properly. Your lumber company has 1,200,000 trees. You plan to harvest 7% of the trees each year. How many years will it take to harvest half of the trees?**

Know
- Number of trees
- Rate of decay

Need
Number of years it takes to harvest 600,000 trees

Plan
- Write an exponential equation.
- Use logarithms to solve the equation.

Think
What equation should you use to model this situation?
Since you are planning to harvest 7% of the trees each year, you should use $y = ab^x$, where b is the decay factor.

Step 1 Is an exponential model reasonable for this situation?

Yes, you are harvesting a fixed percentage each year.

Step 2 Define the variables and determine the model.

Let n = the number of years it takes to harvest half of the trees.

Let $T(n)$ = the number of trees remaining after n years.

A reasonable model is $T(n) = a(b)^n$.

Step 3 Use the model to write an exponential equation.

$T(n) = 600{,}000$

$a = 1{,}200{,}000$

$r = -7\% = -0.07$

$b = 1 + r = 1 + (-0.07) = 0.93$

So, $1{,}200{,}000(0.93)^n = 600{,}000$.

Step 4 Solve the equation. Use logarithms.

$$1{,}200{,}000(0.93)^n = 600{,}000$$

$0.93^n = \frac{600{,}000}{1{,}200{,}000}$ Isolate the term with n.

$\log 0.93^n = \log 0.5$ Take the logarithm of each side.

$n \log 0.93 = \log 0.5$ Power Property of Logarithms.

$n = \frac{\log 0.5}{\log 0.93}$ Solve for n.

$n \approx 9.55$ Use a calculator.

It will take about 9.55 years to harvest half of the original trees.

Got It? **4.** After how many years will you have harvested half of the trees if you harvest 5% instead of 7% yearly?

A **logarithmic equation** is an equation that includes one or more logarithms involving a variable.

Problem 6 Using Logarithmic Properties to Solve an Equation

What is the solution of $\log(x - 3) + \log x = 1$?

$\log(x - 3) + \log x = 1$	
$\log((x - 3)x) = 1$	Product Property of Logarithms
$(x - 3)x = 10^1$	Write in exponential form.
$x^2 - 3x - 10 = 0$	Simplify to a quadratic equation in standard form.
$(x - 5)(x + 2) = 0$	Factor the trinomial.
$x = 5$ or $x = -2$	Solve for x.

Think

What is the domain of the logarithm function?
Logs are defined only for positive numbers. The log of a negative number is undefined.

Check

$\log(x - 3) + \log(x) = 1$

$\log(-2 - 3) + \log(-2) \stackrel{?}{=} 1$ ✘

$\log(x - 3) + \log(x) = 1$

$\log(5 - 3) + \log(5) \stackrel{?}{=} 1$

$\log 2 + \log 5 \stackrel{?}{=} 1$

$0.3010 + 0.6990 = 1$ ✔

If $\log(x - 3) + \log(x) = 1$, $x = 5$.

Got It? **6.** What is the solution of $\log 6 - \log 3x = -2$?