Practice and Problem-Solving Exercises

A Practice

Write each expression as a single logarithm. See Problem 1.

9. $\log 7 + \log 2$

10. $\log_2 9 - \log_2 3$

11. $5 \log 3 + \log 4$

12. $\log 8 - 2 \log 6 + \log 3$

13. $4 \log m - \log n$

14. $\log 5 - k \log 2$

15. $\log_6 5 + \log_6 x$

16. $\log_7 x + \log_7 y - \log_7 z$

17. $\log_3 4 + \log_3 y + \log_3 8x$

Expand each logarithm. See Problem 2.

18. $\log x^3y^5$

19. $\log_7 49xyz$

20. $\log_b \frac{b}{x}$

21. $\log a^2$

22. $\log_5 \frac{r}{s}$

23. $\log_3 (2x)^2$

24. $\log_3 7(2x - 3)^2$

25. $\log \frac{a^2b^3}{c^4}$

26. $\log_4 5\sqrt{x}$

27. $\log_8 8\sqrt{3a^5}$

28. $\log_5 \frac{25}{x}$

29. $\log 10m^4n^{-2}$

Use the Change of Base Formula to evaluate each expression. See Problem 3.

30. $\log_2 9$

31. $\log_{12} 20$

32. $\log_7 30$

33. $\log_5 10$

34. $\log_4 7$

35. $\log_3 54$

36. $\log_5 62$

37. $\log_3 33$

38. Science The concentration of hydrogen ions in household dish detergent is 10^{-12}. What is the pH level of household dish detergent? See Problem 4.

B Apply

Use the properties of logarithms to evaluate each expression.

39. $\log_2 4 - \log_2 16$

40. $\log_2 96 - \log_2 3$

41. $\log_3 27 - 2\log_3 3$

42. $\log_6 12 + \log_6 3$

43. $\log_4 48 - \frac{1}{2}\log_4 9$

44. $\frac{1}{2}\log_5 15 - \log_5 \sqrt{75}$

45. Think About a Plan The loudness in decibels (dB) of a sound is defined as $10 \log \frac{I}{I_0}$, where I is the intensity of the sound in watts per square meter (W/m^2). I_0, the intensity of a barely audible sound, is equal to $10^{-12}\ \text{W/m}^2$. Town regulations require the loudness of construction work not to exceed 100 dB. Suppose a construction team is blasting rock for a roadway. One explosion has an intensity of $1.65 \times 10^{-2}\ \text{W/m}^2$. Is this explosion in violation of town regulations?

- Which physical value do you need to calculate to answer the question?
- What values should you use for I and I_0?

46. Construction The foreman of a construction team puts up a sound barrier that reduces the intensity of the noise by 50%. By how many decibels is the noise reduced? Use the formula $L = 10 \log \frac{I}{I_0}$ to measure loudness. (*Hint:* Find the difference between the expression for loudness for intensity I and the expression for loudness for intensity $0.5I$.)

47. Error Analysis Explain why the expansion at the right of $\log_4 \sqrt{\frac{t}{s}}$ is incorrect. Then do the expansion correctly.

$$\log_4 \sqrt{\frac{t}{s}} = \frac{1}{2} \log_4 \frac{t}{s}$$
$$= \frac{1}{2} \log_4 t - \log_4 s$$

48. Reasoning Can you expand $\log_3 (2x + 1)$? Explain.

49. Writing Explain why $\log (5 \cdot 2) \neq \log 5 \cdot \log 2$.

Determine if each statment is *true* or *false*. Justify your answer.

50. $\log_2 4 + \log_2 8 = 5$

51. $\log_3 \frac{3}{2} = \frac{1}{2}\log_3 3$

52. $\log(x - 2) = \dfrac{\log x}{\log 2}$

53. $\dfrac{\log_b x}{\log_b y} = \log_b \frac{x}{y}$

54. $(\log x)^2 = \log x^2$

55. $\log_4 7 - \log_4 3 = \log_4 4$

Write each logarithmic expression as a single logarithm.

56. $\frac{1}{4}\log_3 2 + \frac{1}{4}\log_3 x$

57. $\frac{1}{2}(\log_x 4 + \log_x y) - 3\log_x z$

58. $x\log_4 m + \frac{1}{y}\log_4 n - \log_4 p$

59. $\left(\dfrac{2\log_b x}{3} + \dfrac{3\log_b y}{4}\right) - 5\log_b z$

Expand each logarithm.

60. $\log\sqrt{\dfrac{2x}{y}}$

61. $\log\dfrac{s\sqrt{7}}{t^2}$

62. $\log\left(\dfrac{2\sqrt{x}}{5}\right)^3$

63. $\log\dfrac{m^3}{n^4p^{-2}}$

64. $\log 4\sqrt{\dfrac{4r}{s^2}}$

65. $\log_b \dfrac{\sqrt{x}\sqrt[3]{y^2}}{\sqrt[5]{z^2}}$

66. $\log_4 \dfrac{\sqrt{x^5y^7}}{zw^4}$

67. $\log\dfrac{\sqrt{x^2 - 4}}{(x + 3)^2}$

Write each logarithm as the quotient of two common logarithms. Do not simplify the quotient.

68. $\log_7 2$

69. $\log_3 8$

70. $\log_5 140$

71. $\log_9 3.3$

72. $\log_4 3x$