7-4 Properties of Logarithms

Objective To use the properties of logarithms

take note

Properties Properties of Logarithms

For any positive numbers m, n, and b where $b \neq 1$, the following properties apply.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

ONLINE PROBLEMS

Problem 1 Simplifying Logarithms

What is each expression written as a single logarithm?

A $\log_4 32 - \log_4 2$

$\log_4 32 - \log_4 2 = \log_4 \frac{32}{2}$ Quotient Property of Logarithms

$= \log_4 16$ Divide.

$= \log_4 4^2$ Write 16 as a power of 4.

$= 2$ Simplify.

B $6 \log_2 x + 5 \log_2 y$

$6 \log_2 x + 5 \log_2 y = \log_2 x^6 + \log_2 y^5$ Power Property of Logarithms

$= \log_2 x^6 y^5$ Product Property of Logarithms

Think

What must you do with the numbers that multiply the logarithms?
Apply the Power Property of Logarithms.

Got It? **1.** What is each expression written as a single logarithm?

a. $\log_4 5x + \log_4 3x$ **b.** $2 \log_4 6 - \log_4 9$

Problem 2 Expanding Logarithms

What is each logarithm expanded?

A $\log \frac{4x}{y}$

$$\log \frac{4x}{y} = \log 4x - \log y \qquad \text{Quotient Property of Logarithms}$$
$$= \log 4 + \log x - \log y \qquad \text{Product Property of Logarithms}$$

B $\log_9 \frac{x^4}{729}$

Think

Can you apply the Power Property of Logarithms first?
No; the fourth power applies only to x.

$$\log_9 \frac{x^4}{729} = \log_9 x^4 - \log_9 729 \qquad \text{Quotient Property of Logarithms}$$
$$= 4 \log_9 x - \log_9 729 \qquad \text{Power Property of Logarithms}$$
$$= 4 \log_9 x - \log_9 9^3 \qquad \text{Write 729 as a power of 9.}$$
$$= 4 \log_9 x - 3 \qquad \text{Simplify.}$$

Got It? **2.** What is each logarithm expanded?

a. $\log_3 \frac{250}{37}$ **b.** $\log_3 9x^5$

You have seen logarithms with many bases. The log key on a calculator finds $\log_{10}$ of a number. To evaluate a logarithm with any base, use the **Change of Base Formula**.

take note

Property Change of Base Formula

For any positive numbers m, b, and c, with $b \neq 1$ and $c \neq 1$,

$$\log_b m = \frac{\log_c m}{\log_c b}.$$

Here's Why It Works

$$\log_b m = \frac{(\log_b m)(\log_c b)}{\log_c b} \qquad \text{Multiply } \log_b m \text{ by } \frac{\log_c b}{\log_c b} = 1.$$
$$= \frac{\log_c b^{\log_b m}}{\log_c b} \qquad \text{Power Property of Logarithms}$$
$$= \frac{\log_c m}{\log_c b} \qquad b^{\log_b m} = m$$

Problem 3 Using the Change of Base Formula

What is the value of each expression?

Think

What common base has powers that equal 27 and 81?
3; $3^3 = 27$ and $3^4 = 81$.

A $\log_{81} 27$

Method 1 Use a common base.

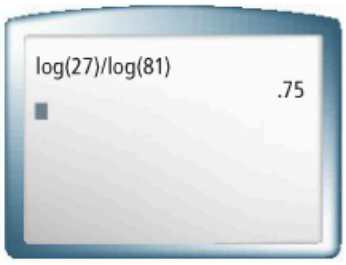
$$\log_{81} 27 = \frac{\log_3 27}{\log_3 81}$$ Change of Base Formula

$$= \frac{3}{4}$$ Simplify.

Method 2 Use a calculator.

$$\log_{81} 27 = \frac{\log 27}{\log 81}$$ Change of Base Formula

$$= 0.75$$ Use a calculator.



Think

What would be a reasonable result?
$5^2 = 25$ and $5^3 = 125$, so $\log_5 36$ should be between 2 and 3.

B $\log_5 36$

$$\log_5 36 = \frac{\log 36}{\log 5}$$ Change of Base Formula

$$\approx 2.23$$ Use a calculator to evaluate.

Got It? **3.** Use the Change of Base Formula. What is the value of each expression?

a. $\log_8 32$ **b.** $\log_4 18$

Chemistry The pH of a substance equals $-\log [H^+]$, where $[H^+]$ is the concentration of hydrogen ions. $[H^+_a]$ for household ammonia is 10^{-11}. $[H^+_v]$ for vinegar is 6.3×10^{-3}. What is the difference of the pH levels of ammonia and vinegar?

Think	Write
Write the equation for pH.	$pH = -\log [H^+]$
Write the difference of the pH levels.	$-\log [H^+_a] - (-\log [H^+_v])$
	$= -\log [H^+_a] + \log [H^+_v]$
	$= \log [H^+_v] - \log [H^+_a]$
Substitute values for $[H^+_v]$ and $[H^+_a]$.	$= \log (6.3 \times 10^{-3}) - \log 10^{-11}$
Use the Product Property of Logarithms, and simplify.	$= \log 6.3 + \log 10^{-3} - \log 10^{-11}$
	$= \log 6.3 - 3 + 11$
Use a calculator.	≈ 8.8
Write the answer.	The pH level of ammonia is about 8.8 greater than the pH level